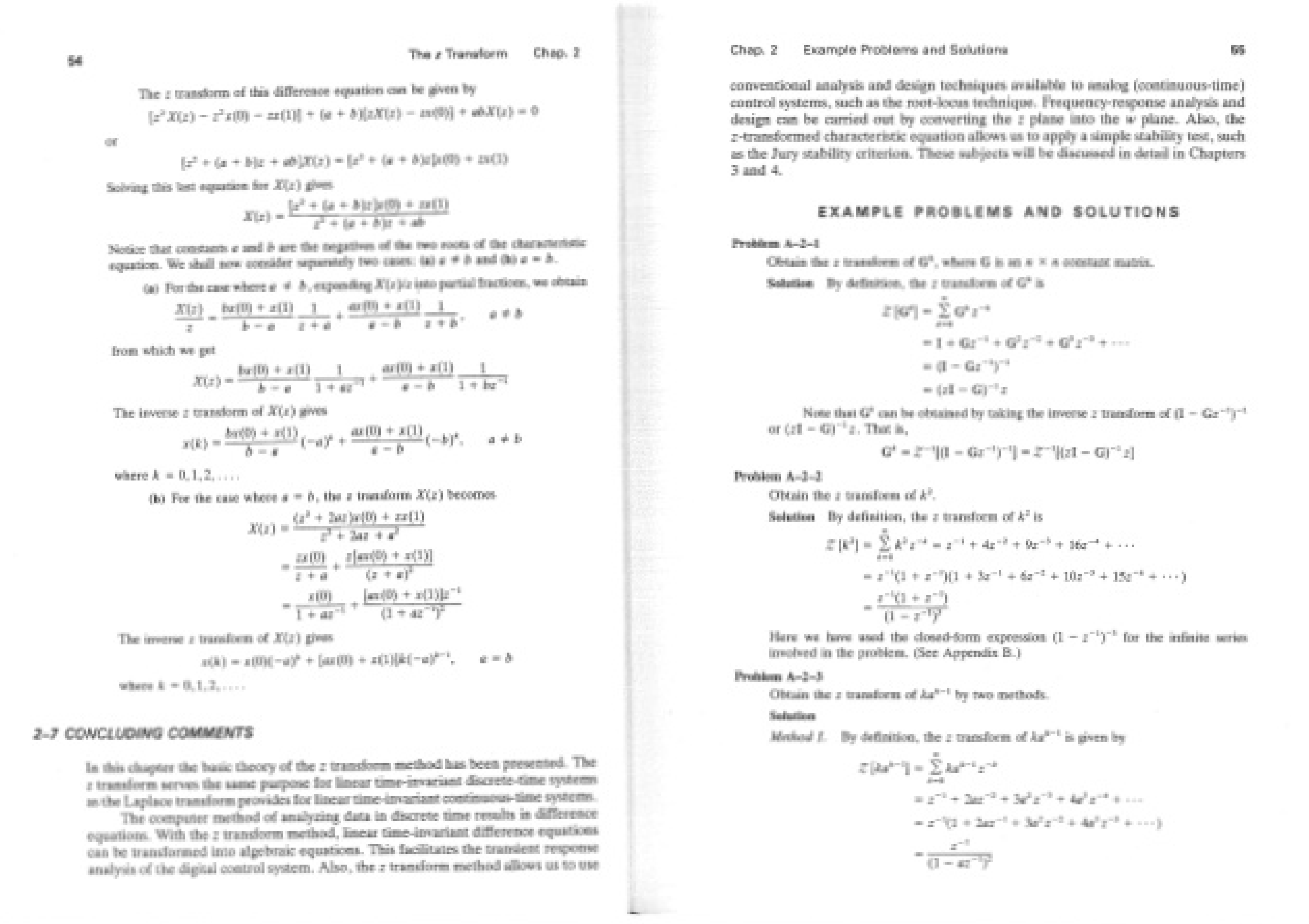

The z transform of this difference equation can be given by

$$[z^2X(z) - z^2x(0) - zx(1)] + (a + b)[zX(z) - zx(0)] + abX(z) = 0$$

or

$$[z^2 + (a + b)z + ab]X(z) = [z^2 + (a + b)z]x(0) + zx(1)$$

Solving this last equation for $X(z)$ gives

$$X(z) = \frac{[z^2 + (a + b)z]x(0) + zx(1)}{z^2 + (a + b)z + ab}$$

Notice that constants a and b are the negatives of the two roots of the characteristic equation. We shall now consider separately two cases: (a) $a \neq b$ and (b) $a = b$.

(a) For the case where $a \neq b$, expanding $X(z)/z$ into partial fractions, we obtain

$$\frac{X(z)}{z} = \frac{bx(0) + x(1)}{b - a}\frac{1}{z + a} + \frac{ax(0) + x(1)}{a - b}\frac{1}{z + b}, \qquad a \neq b$$

from which we get

$$X(z) = \frac{bx(0) + x(1)}{b - a}\frac{1}{1 + az^{-1}} + \frac{ax(0) + x(1)}{a - b}\frac{1}{1 + bz^{-1}}$$

The inverse z transform of $X(z)$ gives

$$x(k) = \frac{bx(0) + x(1)}{b - a}(-a)^k + \frac{ax(0) + x(1)}{a - b}(-b)^k, \qquad a \neq b$$

where $k = 0, 1, 2, \ldots$.

(b) For the case where $a = b$, the z transform $X(z)$ becomes

$$X(z) = \frac{(z^2 + 2az)x(0) + zx(1)}{z^2 + 2az + a^2}$$
$$= \frac{zx(0)}{z + a} + \frac{z[ax(0) + x(1)]}{(z + a)^2}$$
$$= \frac{x(0)}{1 + az^{-1}} + \frac{[ax(0) + x(1)]z^{-1}}{(1 + az^{-1})^2}$$

The inverse z transform of $X(z)$ gives

$$x(k) = x(0)(-a)^k + [ax(0) + x(1)]k(-a)^{k-1}, \qquad a = b$$

where $k = 0, 1, 2, \ldots$.

2-7 CONCLUDING COMMENTS

In this chapter the basic theory of the z transform method has been presented. The z transform serves the same purpose for linear time-invariant discrete-time systems as the Laplace transform provides for linear time-invariant continuous-time systems.

The computer method of analyzing data in discrete time results in difference equations. With the z transform method, linear time-invariant difference equations can be transformed into algebraic equations. This facilitates the transient response analysis of the digital control system. Also, the z transform method allows us to use

conventional analysis and design techniques available to analog (continuous-time) control systems, such as the root-locus technique. Frequency-response analysis and design can be carried out by converting the z plane into the w plane. Also, the z-transformed characteristic equation allows us to apply a simple stability test, such as the Jury stability criterion. These subjects will be discussed in detail in Chapters 3 and 4.

EXAMPLE PROBLEMS AND SOLUTIONS

Problem A-2-1

Obtain the z transform of $\mathbf{G}^k$, where $\mathbf{G}$ is an $n \times n$ constant matrix.

Solution By definition, the z transform of $\mathbf{G}^k$ is

$$\mathcal{Z}[\mathbf{G}^k] = \sum_{k=0}^{\infty} \mathbf{G}^k z^{-k}$$
$$= \mathbf{I} + \mathbf{G}z^{-1} + \mathbf{G}^2z^{-2} + \mathbf{G}^3z^{-3} + \cdots$$
$$= (\mathbf{I} - \mathbf{G}z^{-1})^{-1}$$
$$= (z\mathbf{I} - \mathbf{G})^{-1}z$$

Note that $\mathbf{G}^k$ can be obtained by taking the inverse z transform of $(\mathbf{I} - \mathbf{G}z^{-1})^{-1}$ or $(z\mathbf{I} - \mathbf{G})^{-1}z$. That is,

$$\mathbf{G}^k = \mathcal{Z}^{-1}[(\mathbf{I} - \mathbf{G}z^{-1})^{-1}] = \mathcal{Z}^{-1}[(z\mathbf{I} - \mathbf{G})^{-1}z]$$

Problem A-2-2

Obtain the z transform of k^2.

Solution By definition, the z transform of k^2 is

$$\mathcal{Z}[k^2] = \sum_{k=0}^{\infty} k^2z^{-k} = z^{-1} + 4z^{-2} + 9z^{-3} + 16z^{-4} + \cdots$$
$$= z^{-1}(1 + z^{-1})(1 + 3z^{-1} + 6z^{-2} + 10z^{-3} + 15z^{-4} + \cdots)$$
$$= \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}$$

Here we have used the closed-form expression $(1 - z^{-1})^{-3}$ for the infinite series involved in the problem. (See Appendix B.)

Problem A-2-3

Obtain the z transform of ka^{k-1} by two methods.

Solution

Method 1. By definition, the z transform of ka^{k-1} is given by

$$\mathcal{Z}[ka^{k-1}] = \sum_{k=0}^{\infty} ka^{k-1}z^{-k}$$
$$= z^{-1} + 2az^{-2} + 3a^2z^{-3} + 4a^3z^{-4} + \cdots$$
$$= z^{-1}(1 + 2az^{-1} + 3a^2z^{-2} + 4a^3z^{-3} + \cdots)$$
$$= \frac{z^{-1}}{(1 - az^{-1})^2}$$

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